Engineering Notes

Wingtip Vortex Simulation by Using Nonequilibrium Eddy Viscosity Model

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Nomenclature

c	=	wing chord
C_p	=	pressure coefficient
k	=	turbulent kinetic energy
P_k	=	production rate of turbulent kinetic energy
U_i	=	velocity vector
x, y, z	=	streamwise, normal, and spanwise coordinates with origin at the quarter chord
x'	=	streamwise coordinate measured from the trailing edge
ε	=	dissipation rate

I. Introduction

WINGTIP vortex is an important subject in fluid mechanics, as it is involved in many practical engineering problems that can be found, for example, in [1–4]. Thus, it has long been the subject of numerous experimental and/or numerical investigations. Chigier and Corsiglia [5] pioneered the experimental study of the wingtip vortex. They measured, by using hot-wire anemometry, the wingtip vortex of a square-tipped rectangular wing up to a $12c$ downstream distance. The maximum axial velocity at the vortex core at $x'/c = -0.25$ was $1.4U_\infty$. They showed that the maximum tangential velocity increases with the angle of attack, while the vortex core radius remains unchanged. Green [6] and Green and Acosta [7] used double-pulsed holography to measure the instantaneous velocity distribution in trailing vortices of a round-tipped rectangular wing. At a 10 deg angle of attack, the axial velocity at the vortex core was $1.6U_\infty$ at $x'/c = 2.0$. Chow et al. [8] used hot-wire and seven-hole probes to measure the flow over and immediately downstream of a round-tipped rectangular wing with a NACA0012 section at $Re_c = 4.6 \times 10^6$ and a 10 deg angle of attack. The measurement data of this study will be used as reference data in this work.

Numerical studies began to appear in the late 1980s. Srinivasan et al. [9] used a thin-layer Navier–Stokes solver with the Baldwin–Lomax turbulence model [10] to examine the influence of the tip-cap shape and tip planform on the wingtip vortex. Their results showed good agreement with the experimental data of Spivey and Moorehouse [11] for the surface pressures, except for the surface pressure suction peak induced by the vortex in the vicinity of the wingtip. The computed vorticity contours of the tip vortex, however, were rather poor in the sense that they were highly distorted and more diffusive with the downstream distance. Dacles-Mariani and Zilliac [12] studied, both numerically and experimentally, the wingtip vortex in the near field in conjunction with Chow et al.'s experimental study [8]. The Baldwin–Barth turbulence model [13] was used with the modified production term suggested by Spalart through private communication [12] to reduce the eddy viscosity at the vortex core. The predicted velocity profile was in good agreement with the measured data, but the core static pressure was underpredicted up to 25% at the downstream boundary. Craft et al. [14] recently performed a numerical simulation of the experiment of Chow et al. [8]. Three turbulence models, the eddy viscosity model, the nonlinear eddy viscosity model of Suga [15], and the two-component-limit (TCL) second-moment closure model [16], were adopted in their simulation. The results obtained using linear and nonlinear eddy viscosity models showed a far too rapid decay of the vortex core. Only the TCL model successfully reproduced the principal features of the vortex flowfield.

The TCL model is a Reynolds stress transport model, which is much more complicated for application to practical flows than the two-equation models in spite of its distinguished merits. It is well known that conventional two-equation models perform rather poorly for flows that are not in near-equilibrium flow. To remedy this situation, Yoshizawa et al. [17] suggested a nonequilibrium eddy viscosity model, which works well for swirling (or strong vorticity) flows. The motivation of the present work is to investigate whether this nonequilibrium model performs well for wingtip vortex flow in comparison with the TCL model, which is a first attempt to the authors' knowledge. It is found that the model performs almost as good as the TCL model; hence, it is strongly recommended for practical flow computations of wingtip vortex flow.

II. Numerical Method

A. Nonequilibrium Eddy Viscosity Model

As has been mentioned, Yoshizawa et al. [17] suggested a nonequilibrium eddy viscosity model to take into account a nonstationary or nonequilibrium effect in the time of turbulence. We summarize here only the final model equations proposed. Details of the model are referred to in the original paper [17]. The eddy viscosity ν_T is given as

$$\nu_T = \frac{1}{\Lambda + (C_A/k)(\mathrm{D}/\mathrm{D}t)(k^2/\varepsilon)}\nu_{TE}, \qquad \left(\text{when } \frac{1}{k}\frac{\mathrm{D}}{\mathrm{D}t}\frac{k^2}{\varepsilon} > 0\right) \tag{1}$$

$$\nu_T = \frac{1}{\Lambda}\left(1 - \frac{C_A}{\Lambda}\frac{1}{k}\frac{\mathrm{D}}{\mathrm{D}t}\frac{k^2}{\varepsilon}\right)\nu_{TE}, \qquad \left(\text{when } \frac{1}{k}\frac{\mathrm{D}}{\mathrm{D}t}\frac{k^2}{\varepsilon} < 0\right) \tag{2}$$

where

$$\Lambda = \sqrt{1 + C_S\left(\frac{k}{\varepsilon}S_{ij}\right)^2 + C_\Omega\left(\frac{k}{\varepsilon}\Omega_{ij}\right)^2} \tag{3}$$

Here, $\nu_{TE} = C_\mu k^2/\varepsilon$ is the equilibrium turbulent viscosity, $S_{ij} = (\partial U_j/\partial x_i + \partial U_i/\partial x_j)$ is the mean strain-rate tensor, and $\Omega_{ij} = (\partial U_j/\partial x_i - \partial U_i/\partial x_j)$ is the mean rotational rate tensor.

The two equations for k and ε are the same as those of the standard k-ε model:

$$\frac{\mathrm{D}k}{\mathrm{D}t} = P_k - \varepsilon + \nabla\cdot\left(\frac{\nu_t}{\sigma_k}\nabla\cdot k\right) \tag{4}$$

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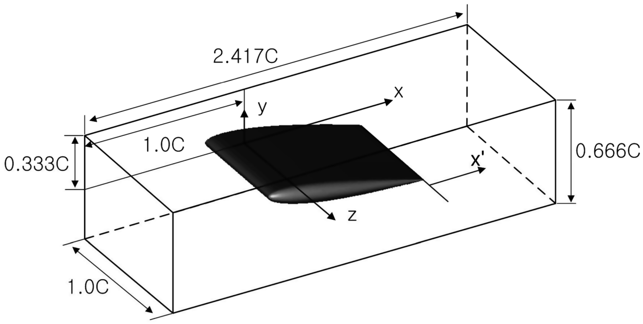

Fig. 1 Computational domain for the simulation of the wingtip vortex.

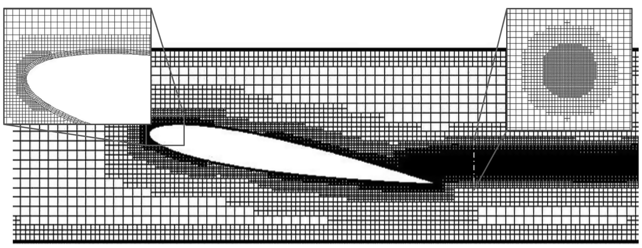

Fig. 2 Computational domain for the simulation of the wingtip vortex.

$$\frac{D\varepsilon}{Dt} = -C_{\varepsilon 1}\frac{\varepsilon}{k}P_k - C_{\varepsilon 2}\frac{\varepsilon^2}{k} + \nabla\cdot\left(\frac{\nu_t}{\sigma_\varepsilon}\nabla\cdot\varepsilon\right) \tag{5}$$

The model constants are

$$C_\mu = 0.09, \quad C_S = 0.060, \quad C_W = 0.02C_S, \quad C_A = 0.76$$
$$\sigma_k = 1.4, \quad \sigma_\varepsilon = 1.4, \quad C_{\varepsilon 1} = 1.5, \quad C_{\varepsilon 2} = 1.9 \tag{6}$$

As can be seen in Eq. (1), the model modifies the conventional eddy viscosity by synthesizing several time scales. One obvious outcome of this modification is the reduction of eddy viscosity in swirling flows. In the present simulation, the following limiters, suggested by Park and Park [18], were also implemented to prevent unrealistically large values of turbulence variables:

$$\nu_t \leq \sqrt{\frac{2}{3}}\frac{k}{|S|} \tag{7}$$

$$P_k \leq \sqrt{\frac{8}{3}}k|S| \tag{8}$$

where $|S| = \sqrt{1/4(\partial U_j/\partial x_i + \partial U_i/\partial x_j)}$

B. Computational Details

The target flow of the present computation is the experimental test case of Chow et al. [8]. The wing is installed in a $0.666c \times 1.0c$ wind tunnel (see Fig. 1) with the inboard surface of the wing attached to the sidewall of the tunnel. The angle of attack is 10 deg, and the Reynolds number is 4.6×10^6 based on the chord length. The half span of the wing is only $0.75c$.

Computations were carried out by using OpenFOAM®[‡], with the subroutine added for the nonequilibrium eddy viscosity model coded by us. The SIMPLE algorithm was selected. The second-order upwind differencing and the central differencing scheme were adopted for the convection term and the diffusion term, respectively. The computational domain included the wind-tunnel walls, as sketched in Fig. 1. Following Craft et al. [14], the upstream boundary was placed at $x/c = -1.0$. For the initial data for the computation with the nonequilibrium model, we used the results obtained with the standard k-ε model.

Table 1 Grid test

Grid	Max. grid spacing	Min. grid spacing	No. of grids at viscous core region	Approx. no. of grids
1	$0.016c$	$0.004c$	10	800,000
2	$0.008c$	$0.002c$	20	2,000,000
3	$0.004c$	$0.001c$	40	4,200,000

The unstructured grid system of hexahedral-based mesh was used with a special cell layer that consisted of grids normal to the solid surface to resolve the boundary layer well. The normal distance of the first grid from the wall is in the range of $y^+ = 30$–100 to employ the wall function approach for the wall boundary condition. A grid convergence test was performed to assure adequate accuracy. At the near wall and vortex core region, the grid was adaptively refined. Figure 2 shows a typical unstructured grid system used in the present computation. Three grid arrangements, as summarized in Table 1, were tested. Subtitles regarding grid spacing in the table are for the grids just outside of the cell layers covering the wall boundary layer.

Results of the grid test with the nonequilibrium eddy viscosity model are illustrated in Figs. 3 and 4. By comparing Figs. 3 and 4, we find that the chordwise surface pressure distribution is much less sensitive to grid arrangements compared with the axial velocity variations of the vortex core along the downstream distance. However, we would like to comment here on the pressure distribution

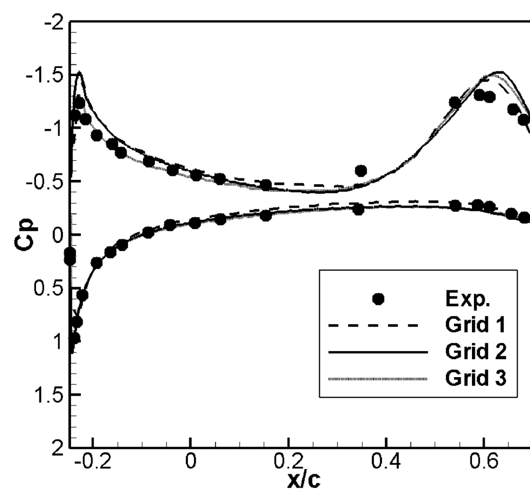

Fig. 3 Chordwise surface pressure distribution at $z/c = 0.667$ (grid test).

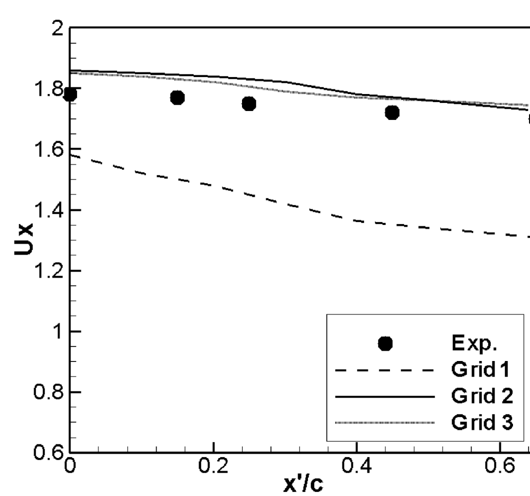

Fig. 4 Variation of peak axial velocity at vortex core (grid test).

[‡]Data available at http://www.openfoam.com/ [retrieved 24 June 2010].

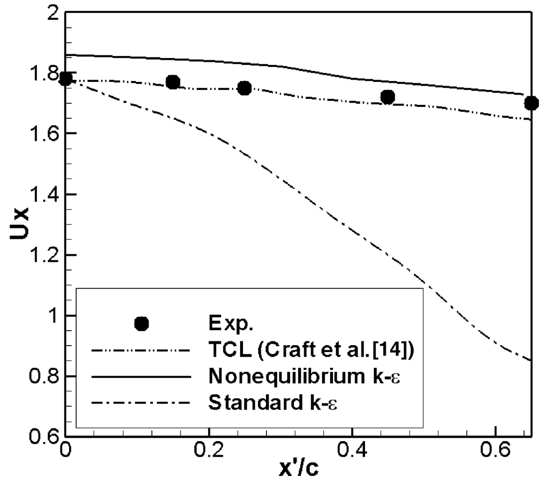

Fig. 5 Peak axial velocity at vortex core.

of Fig. 3. We clearly see a discrepancy between the predictions and the experimental data on the upper surface pressure distributions near the region of the pressure peak. This could also be easily noticed in Dacles-Mariani and Zillac [12]. We further found from our numerical results that the chordwise variation of the pressure distribution was the sensitive location (z/c) near the wing tip: ΔC_P of the peak value was as large as 0.1 when $\Delta\ (z/c)$ was only $0.002c$. Figure 4 suggests that we also need at least 20 cells in each of the crosswise and spanwise directions to properly resolve vortex core flow. Based on the grid test, we use grid 2 for later calculations.

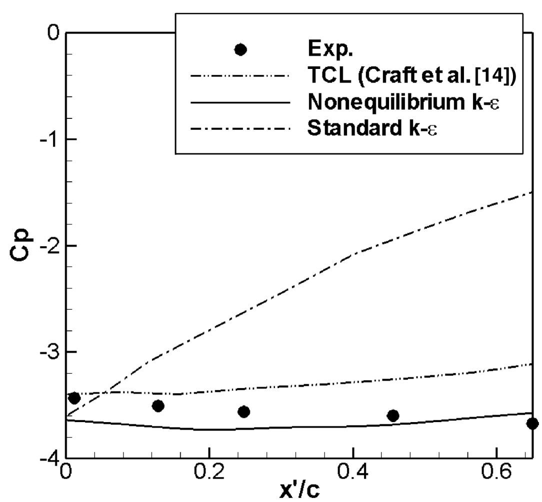

Fig. 6 Static pressure at vortex core.

III. Results and Discussion

Figure 5 shows the variations of the axial velocity (normalized by the freestream velocity) at the vortex core, predicted by using different turbulence models. The standard k-ε model results in a too rapid decrease of the axial velocity, yielding the velocity below the freestream velocity in a downstream distance shorter than a chord length. We see that the nonequilibrium eddy viscosity model and the TCL model predict the very slow decreasing tendency of the vortex core axial velocity in the very near-wake region well. We also see that the velocity magnitude of the present nonequilibrium model case is slightly higher than the experimental data, while the rate of decrease is as good as the TCL model case. Figure 6 shows the static pressure variations at the vortex core along the streamwise direction. We find that the static pressure variation reflects the axial velocity distribution of Fig. 5. The standard k-ε model predicts the static pressure with a very high increasing rate with the downstream distance, while the nonequilibrium and TCL models yield the static pressure values in the same neighborhood of the experimental data. We plot the normalized velocity magnitude distribution across the vortex core at several downstream stations in Fig. 7. The experimental data were only available at $x/c = 1.241$. The velocity magnitude is the root-mean-square value of the axial and tangential velocity components. Figure 7 clearly indicates that the standard k-ε model does not adequately predict the velocity distribution, while the non-equilibrium model performs very well.

IV. Conclusions

This study investigates the performance of the nonequilibrium eddy viscosity model, proposed by Yoshizawa et al. [17], for the simulation of the wingtip vortex. Results predicted by using the standard k-ε model, the TCL model [14], and the nonequilibrium model were compared with the experimental data of Chow et al. [8]. Through the comparison, we found that the nonequilibrium model performed as good as or only slightly inferior to the TCL model. We thus conclude that the nonequilibrium eddy viscosity model is indeed a very good model for practical wingtip vortex simulation, considering the fact that the TCL model is a Reynolds stress transport model, which is not as easy as two-equation turbulence models for practical application.

Acknowledgment

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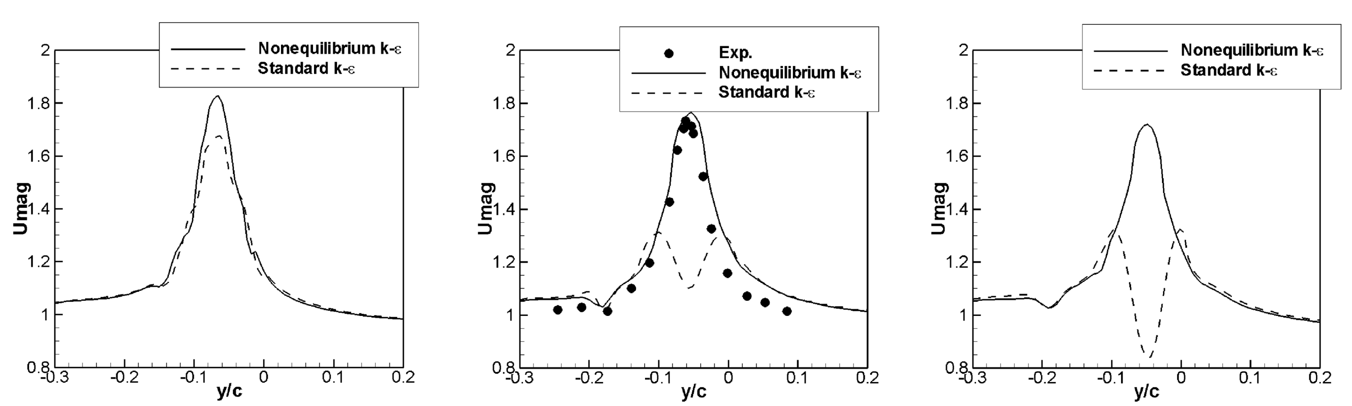

a) x'/c=0.15 b) x'/c=0.54 c) x'/c=0.62

Fig. 7 Velocity magnitude distribution across vortex core.

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